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COMPARISON OF FLUTTER CALCULATIONS USING VARIOUS AERODYNAMIC COEFFICIENTS WITH EXPERIMENTAL RESULTS FOR SOME RECTANGULAR CANTILEVER WINGS AT MACH NUMBER 1.3

By Herbert C. Nelson and Ruby A. Rainey

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

A general Rayleigh analysis is used as a basis for developing four methods of flutter analysis that are applied to twelve low-aspect-ratio wings. These wings were previously tested at a Mach number of 1.3 by progressively varying certain wing parameters until flutter occurred. They were rectangular in plan form and had aspect ratios between 3.00 and 4.55. The four methods of flutter analysis used are: section coefficients for harmonically pitching and translating rectangular wings in a Rayleigh type of analysis, two-dimensional coefficients in a Rayleigh type of analysis, total coefficients for harmonically pitching and translating rectangular wings in a representative-section analysis, and two-dimensional coefficients in a representative-section analysis. Each of the four methods involved two degrees of freedom, namely, first bending and first torsion of a cantilever wing.

The analytical results are compared with the previously obtained experimental values. The comparison indicates that the use of section aerodynamic coefficients derived on the basis of three-dimensional flow leads to a significant improvement in the correlation of theory and experiment.

INTRODUCTION

The problem of theoretically determining the flutter characteristics of unswept wings of low aspect ratio in supersonic flow has become of increased interest. Most of the previous analytical work on this problem has been based on air-force and moment coefficients for two-dimensional supersonic flow, such as those tabulated in reference 1. For example, reference 2 presents the results obtained at a Mach number of 1.3, by using two-dimensional coefficients in a representative-section type of flutter analysis, for twelve unswept wings with aspect ratios ranging from 3.00 to 4.55. As explained in reference 2, these wings were also tested at a Mach number of 1.3 by progressively shifting their centers of gravity and elastic axes and modifying their bending and torsional frequencies until flutter occurred. A comparison of the calculated and experimental results showed that in the majority of cases the calculated flutter speeds were considerably below the experimental flutter speed. This discrepancy suggests in part that, at least in the low supersonic speed range, two-dimensional coefficients are inadequate and more realistic aerodynamic coefficients should be used in the flutter analysis of unswept low-aspect-ratio wings.

In reference 3, streamwise section and total air-force and moment coefficients expanded to the seventh power of the frequency of oscillation were developed for harmonically pitching and translating rectangular wings moving at supersonic speed. The section coefficients were used in a Rayleigh type of flutter analysis to calculate the flutter speeds of a rectangular wing of aspect ratio 4.53 at several Mach numbers in the low supersonic speed range. For comparison the wing was also analyzed by using the two-dimensional coefficients of reference 1 in a Rayleigh type of analysis. Examination of the results showed the flutter speeds based on the rectangular-wing section coefficients to be higher than those based on two-dimensional coefficients, particularly at the lower Mach numbers. Application of a Rayleigh type of analysis involving the section coefficients of reference 3 to the wings of reference 2 might therefore be expected to yield a better correlation between theory and experiment than was obtained in reference 2.

Also of interest is reference 4 in which a comparison is made between flutter results obtained by using two-dimensional coefficients in a representative-section type of analysis and total coefficients for rectangular wings in the same type of analysis. For wing parameters in the range of those given in reference 2, reference 4 also shows an increase in calculated flutter speed resulting from the use of finite-wing coefficients.

In the present paper four methods of analysis are applied to the twelve wings of reference 2 and the results are compared with the experimental results in reference 2. These four methods of flutter analysis are: section coefficients for a pitching and translating rectangular wing in a Rayleigh type of analysis, two-dimensional coefficients in a Rayleigh type of analysis, total coefficients for a pitching and translating rectangular wing in a representative-section type of analysis, and two-dimensional coefficients in a representative-section type of analysis.

SYMBOLS

A aspect ratio, s/b

b one-half chord

c speed of sound in undisturbed medium

g_h, g_α first bending and first torsion damping coefficients, respectively (see ch. IX of ref. 5)

h vertical displacement of axis of rotation x_o, positive downward

$\bar{h}$ generalized coordinate in bending degree of freedom, $h_o e^{i\omega t}$

h_o bending amplitude at tip of wing

k reduced frequency, $\omega b/V$

l_h, m_h coefficients of section lift and moment, respectively, associated with mode shape Z_h

l_α, m_α coefficients of section lift and moment, respectively, associated with mode shape Z_α

L_i, M_i components of section force and moment coefficients, respectively, for rectangular wing (see ref. 3) in equation (7) and for two-dimensional wing (see ref. 1) in equation (8); $i = 1, 2, 3$, and 4

$\bar{L}_i, \bar{M}_i$ components of total force and moment coefficients, respectively, for rectangular wing (see ref. 3); $i = 1, 2, 3$, and 4

M Mach number, V/c

M_α aerodynamic section moment on wing about axis of rotation x_o, positive leading edge up

P aerodynamic section normal force, positive downward

r_α nondimensional radius of gyration of wing section about elastic axis, $\sqrt{I_\alpha/mb^2}$ where I_α is mass moment of inertia per unit span about elastic axis and m is mass of wing per unit span

s	one-half span of wing
t	time
V	velocity of flow
x	nondimensional chordwise coordinate measured from leading edge, referred to wing chord $2b$
x_α	location of center of gravity of wing measured from elastic axis (see ref. 1)
x_o	chordwise position of axis of rotation of wing (elastic axis)
y	nondimensional spanwise coordinate measured from midspan of wing, referred to wing half-span s
Z_h	first bending mode shape of wing
Z_α	first torsion mode shape of wing
α	angle of attack, positive leading edge up
$\bar{\alpha}$	generalized coordinate in torsion degree of freedom, $\alpha_o e^{i\omega t}$
α_o	torsion amplitude at tip of wing
$\beta = \sqrt{M^2 - 1}$	
κ	density parameter, $\pi\rho b^2/m$
ρ	density in undisturbed medium
ω	frequency of oscillation at flutter
ω_h	first bending frequency of wing
ω_α	first torsion frequency of wing

METHODS OF FLUTTER ANALYSIS

Rayleigh Analysis

<u>General considerations</u>.- The wings to be analyzed are rectangular in plan form and were tested as cantilevers in the Langley supersonic

flutter apparatus (a 9-inch by 18-inch supersonic drawdown tunnel). In a Rayleigh type of analysis of such wings, the bending component of the flutter mode can be approximated by the first bending mode of a uniform cantilever wing and the torsion component by the first torsion mode. The flutter determinant is then formed and is solved for the flutter condition. (A detailed discussion of the Rayleigh type of analysis as applied to flutter may be found in ch. IX of ref. 5.)

The bending component h and the torsion component α of the flutter mode may be written as

$$\left.\begin{aligned} h(y,t) &= Z_h(y)\bar{h}(t) \\ \alpha(y,t) &= Z_\alpha(y)\bar{\alpha}(t) \end{aligned}\right\} \tag{1}$$

where y is the nondimensional coordinate shown in figure 1, Z_h and Z_α are the first bending and first torsion mode shapes shown in figure 2, and $\bar{h}$ and $\bar{\alpha}$ are the generalized coordinates in the bending and torsion degrees of freedom, respectively. The section aerodynamic force or aerodynamic force per unit span, positive downward, associated with equations (1) may be written as

$$P = -4\rho b^2\omega^2\left[l_h(y)\bar{h} + l_\alpha(y)\bar{\alpha}\right] \tag{2}$$

and the section moment, positive leading edge up, about the arbitrary axis of rotation $x = x_o$ may be written as

$$M_\alpha = -4\rho b^3\omega^2\left[m_h(y)\bar{h} + m_\alpha(y)\bar{\alpha}\right] \tag{3}$$

where ω is the frequency of oscillation, b is the one-half chord of the wing, l_h and m_h are complex coefficients of the lift and moment associated with the mode Z_h, and l_α and m_α are the complex coefficients of the lift and moment associated with the mode Z_α. Each of the aerodynamic coefficients l_h, m_h, l_α, and m_α, in addition to being a function of the spanwise variable y, is a function of Mach number M and reduced frequency $k = b\omega/V$. Although these coefficients may be taken to apply at either subsonic or supersonic speed, the present paper is concerned only with the supersonic speed range.

The equilibrium equations at flutter may be obtained by setting up the potential and kinetic energies and the work of the applied forces, introducing equations (1), (2), and (3) and the mass and stiffness properties of the wing, and then applying Lagrange's dynamical equation, as shown in chapter IX of reference 5. From the equilibrium equations a flutter determinant may be obtained in the form

$$\begin{vmatrix} A_{hh} & A_{h\alpha} \\ A_{\alpha h} & A_{\alpha\alpha} \end{vmatrix} = 0 \tag{4}$$

where the determinant elements are

$$\left.\begin{aligned}
A_{hh} &= \left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]\int_0^1 Z_h^2 dy - \frac{4}{\pi}\kappa\int_0^1 l_h Z_h\,dy \\
A_{h\alpha} &= x_\alpha\int_0^1 Z_h Z_\alpha\,dy - \frac{4}{\pi}\kappa\int_0^1 l_\alpha Z_h\,dy \\
A_{\alpha h} &= x_\alpha\int_0^1 Z_\alpha Z_h\,dy - \frac{4}{\pi}\kappa\int_0^1 m_h Z_\alpha\,dy \\
A_{\alpha\alpha} &= r_\alpha^2\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right]\int_0^1 Z_\alpha^2 dy - \frac{4}{\pi}\kappa\int_0^1 m_\alpha Z_\alpha\,dy
\end{aligned}\right\} \tag{5}$$

From equations (4) and (5) four methods of analysis are obtained by using various approximations in evaluating the integrals of equation (5).

<u>Section coefficients for rectangular wing</u>.- The following approximate expressions for the section coefficients are employed:

$$\left.\begin{aligned}
l_h &= Z_h\left(L_1 + iL_2\right) \\
l_\alpha &= Z_\alpha\left(L_3 + iL_4\right) \\
m_h &= Z_h\left(M_1 + iM_2\right) \\
m_\alpha &= Z_\alpha\left(M_3 + iM_4\right)
\end{aligned}\right\} \tag{6}$$

where L_i and M_i $(i = 1, 2, 3, \text{ and } 4)$ are the components of the section coefficients given in reference 3 for a rectangular wing in supersonic flow oscillating harmonically as a rigid body in pitch and vertical translation. (A preliminary unpublished analysis, based on parabolic bending of a rectangular wing which closely resembles the mode shape Z_h, suggests that the results obtained by using the distributions of lift and moment for the mode shapes Z_h and Z_α would be nearly identical to the results obtained by using the approximate distributions given by eqs. (6), when multiplied by the mode shape Z_h or Z_α and integrated in the manner required in eqs. (5).) Upon substituting equations (6) into equations (5), the determinant elements of equation (4) become

$$\left.\begin{aligned}
A_{hh} &= \left[1 - \left(\frac{\omega_h}{\omega}\right)^2\right]\int_0^1 Z_h^2 dy - \frac{4}{\pi}\kappa\int_0^1 \left(L_1 + iL_2\right)Z_h^2 dy \\
A_{h\alpha} &= x_\alpha\int_0^1 Z_h Z_\alpha\, dy - \frac{4}{\pi}\kappa\int_0^1 \left(L_3 + iL_4\right)Z_h Z_\alpha\, dy \\
A_{\alpha h} &= x_\alpha\int_0^1 Z_\alpha Z_h\, dy - \frac{4}{\pi}\kappa\int_0^1 \left(M_1 + iM_2\right)Z_\alpha Z_h\, dy \\
A_{\alpha\alpha} &= r_\alpha^2\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right]\int_0^1 Z_\alpha^2 dy - \frac{4}{\pi}\kappa\int_0^1 \left(M_3 + iM_4\right)Z_\alpha^2 dy
\end{aligned}\right\} \tag{7}$$

The uncoupled first bending mode shape Z_h and the first torsion mode shape Z_α needed for the evaluation of the integrals of equations (7) are shown in figure 2. The integrals of equations (7) containing only mode shapes can be evaluated to give

$$\int_0^1 Z_h^2 dy = 0.25$$

$$\int_0^1 Z_h Z_\alpha \, dy = 0.337$$

$$\int_0^1 Z_\alpha^2 dy = 0.50$$

A numerical method for evaluating the integrals of equation (7) involving the aerodynamic coefficients L_i and M_i is given in appendix B of reference 3. (In using ref. 3, note that the spanwise coordinate y of the present paper and the spanwise coordinate ζ of the reference paper are related by $y = 1 - \zeta$.)

Coefficients for two-dimensional wing.- If two-dimensional air-force and moment coefficients are used in place of the section coefficients of reference 3, the force and moment coefficients in equations (7) appear as constants in the integrals and can be factored from under the integral signs, and the determinant elements of equation (4) become

$$\left.\begin{aligned}
A_{hh} &= \left[1 - \left(\frac{\omega_h}{\omega}\right)^2 - \frac{4}{\pi}\kappa\left(L_1 + iL_2\right)\right]\int_0^1 Z_h^2 dy \\
A_{h\alpha} &= \left[x_\alpha - \frac{4}{\pi}\kappa\left(L_3 + iL_4\right)\right]\int_0^1 Z_h Z_\alpha \, dy \\
A_{\alpha h} &= \left[x_\alpha - \frac{4}{\pi}\kappa\left(M_1 + iM_2\right)\right]\int_0^1 Z_\alpha Z_h \, dy \\
A_{\alpha\alpha} &= \left\{r_\alpha^2\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right] - \frac{4}{\pi}\kappa\left(M_3 + iM_4\right)\right\}\int_0^1 Z_\alpha^2 dy
\end{aligned}\right\} \qquad (8)$$

where L_i and M_i ($i = 1, 2, 3,$ and 4) now refer to components of two-dimensional coefficients, such as those tabulated in reference 1.

Representative-Section Analysis

Total coefficients for rectangular wing.- By applying mean-value theory to the integrals in equation (7) and, in the process, by assuming the representative section to be the same for all integrals involved, the determinant elements can be written as

$$\left.\begin{aligned}
A_{hh} &= \left[1 - \left(\frac{\omega_h}{\omega}\right)^2 - \frac{4}{\pi}\kappa\int_0^1 \left(L_1 + iL_2\right)dy\right]\left(Z_h^2\right)_r \\
A_{h\alpha} &= \left[x_\alpha - \frac{4}{\pi}\kappa\int_0^1 \left(L_3 + iL_4\right)dy\right]\left(Z_h Z_\alpha\right)_r \\
A_{\alpha h} &= \left[x_\alpha - \frac{4}{\pi}\kappa\int_0^1 \left(M_1 + iM_2\right)dy\right]\left(Z_\alpha Z_h\right)_r \\
A_{\alpha\alpha} &= \left\{r_\alpha^2\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right] - \frac{4}{\pi}\kappa\int_0^1 \left(M_3 + iM_4\right)dy\right\}\left(Z_\alpha^2\right)_r
\end{aligned}\right\} \tag{9}$$

where the subscript r denotes evaluation at a representative spanwise station $y = r$. Since the quantities having the subscript r cancel in the solution of equation (4), equations (9) may be rewritten as

$$\left.\begin{aligned}
A_{hh} &= 1 - \left(\frac{\omega_h}{\omega}\right)^2 - \frac{4}{\pi}\kappa\left(\bar{L}_1 + i\bar{L}_2\right) \\
A_{h\alpha} &= x_\alpha - \frac{4}{\pi}\kappa\left(\bar{L}_3 + i\bar{L}_4\right) \\
A_{\alpha h} &= x_\alpha - \frac{4}{\pi}\kappa\left(\bar{M}_1 + i\bar{M}_2\right) \\
A_{\alpha\alpha} &= r_\alpha^2\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right] - \frac{4}{\pi}\kappa\left(\bar{M}_3 + i\bar{M}_4\right)
\end{aligned}\right\} \tag{10}$$

where $\bar{L}_i = \int_0^1 L_i\,dy$ and $\bar{M}_i = \int_0^1 M_i\,dy$ $(i = 1, 2, 3, \text{ and } 4)$ are the components of the total force and moment coefficients for a pitching and translating rectangular wing given in reference 3.

<u>Coefficients for two-dimensional wing</u>.- If infinite aspect ratio is substituted into the aerodynamic coefficients of equation (10) (see ref. 3), the determinant elements can be written as

$$\left.\begin{aligned}
A_{hh} &= 1 - \left(\frac{\omega_h}{\omega}\right)^2 - \frac{4}{\pi}\kappa\left(L_1 + iL_2\right) \\
A_{h\alpha} &= x_\alpha - \frac{4}{\pi}\kappa\left(L_3 + iL_4\right) \\
A_{\alpha h} &= x_\alpha - \frac{4}{\pi}\kappa\left(M_1 + iM_2\right) \\
A_{\alpha\alpha} &= r_\alpha^{\,2}\left[1 - \left(\frac{\omega_\alpha}{\omega}\right)^2\right] - \frac{4}{\pi}\kappa\left(M_3 + iM_4\right)
\end{aligned}\right\} \tag{11}$$

where, as in equation (8), L_i and M_i refer to components of two-dimensional coefficients, such as those tabulated in reference 1.

Solution of Flutter Determinant

The flutter condition is determined from the nontrivial solution of equation (4) obtained by using as determinant elements the various approximate forms of equations (5) given by equations (7), (8), (10), and (11). This condition, which requires that the real and imaginary parts of equation (4) vanish simultaneously for the same set of aerodynamic and wing parameters, may be obtained by various means (see ch. XIII of ref. 5).

In the present paper the ratio ω_h/ω in equation (4) is replaced by the equivalent quantity $\left(\omega_h/\omega_\alpha\right)\left(\omega_\alpha/\omega\right)$. Then, for a particular wing and Mach number, for which values of M, κ, x_o, x_α, r_α^2, and ω_h/ω_α

are specified, equation (4) contains the two unknown parameters ω_α/ω and $k = b\omega/V$. The reduced frequency k (upon which the various aerodynamic terms are dependent) is varied until the same value of ω_α/ω is obtained from both the real and imaginary parts of equation (4). This is the required condition and yields the values of k and ω_α/ω at flutter and consequently the flutter-speed coefficients $V/b\omega_\alpha$ for the wing at the selected value of M.

APPLICATION AND DISCUSSION OF RESULTS

The four methods of analysis outlined in the previous section were applied to the twelve wings of reference 2. The wing parameters needed in these analyses and a description of each wing profile, obtained from reference 2, are given in table I of the present paper. The flutter parameters, $V/b\omega$ and ω/ω_α and consequently $V/b\omega_\alpha$, calculated by these methods are listed in table II. For comparison table II also includes the experimentally determined flutter parameters given in reference 2.

In figure 3 the data of table II are plotted in line-graph form. The line-graph method of plotting is employed to achieve a separation of the data and ease of comparison not otherwise obtained because of the insufficient range of variation of the different wing parameters. Also shown in figure 3, as flagged points, are the analytical results of reference 2. These results were obtained by the last method of the previous section (two-dimensional coefficients in a representative-section analysis) but included structural damping. Structural damping could also have been included in the calculations of the present paper by replacing ω_h^2 by $\omega_h^2\left(1 + ig_h\right)$ and ω_α^2 by $\omega_\alpha^2\left(1 + ig_\alpha\right)$, where g_h is the damping coefficient in bending and g_α is the damping coefficient in torsion, in the methods discussed previously. Since damping was not included, the calculations of reference 2 may serve to indicate the effect the inclusion of damping would have on the calculations of the present paper.

Figure 3(a) shows a comparison for each wing of the values of reduced flutter speed $V/b\omega$ (reciprocal of reduced frequency k), obtained by the four methods of analysis. The results obtained by using finite-wing section coefficients in a Rayleigh analysis are closest to experiment in all twelve of the cases treated.

In figure 3(b) values of the ratio of flutter frequency to torsional frequency ω/ω_α are compared for the various wing models. As may be noted in the figure, for five of the wing models (A-1, B-1, C-1, C-2, and F-1) comparatively large differences between the theoretical and experimental values of ω/ω_α exist; these differences would probably be reduced by the inclusion of more degrees of freedom in the various analyses. However, it may be seen by comparing the flagged and unflagged right triangles in figure 3(b) that the inclusion of structural damping in the Rayleigh analysis involving section coefficients for a rectangular wing may sufficiently reduce the differences between experiment and theory.

Figure 3(c) shows a comparison of the values of flutter-speed coefficient $V/b\omega_\alpha$ determined for the various wing models from the data presented in figures 3(a) and 3(b). The results of using the Rayleigh analysis involving section coefficients for rectangular wings are closest to experiment in the majority of the cases treated, that is, except for models A-1, C-1, and C-2. The section-coefficient results in these cases are above the experimental values (nonconservative). Inclusion of more modes in the analysis may relieve this situation.

Also of interest in the present comparison are the curves of $V/b\omega_\alpha$ calculated in reference 3 for model B-1 of table I in the Mach number range $10/9 \leqq M \leqq 10/6$ by the first two methods of the previous section, that is, section coefficients for a nondeformable rectangular wing in a Rayleigh type of analysis and two-dimensional coefficients in the same type of analysis. These curves, taken from figure 12 of reference 3, are shown in figure 4. The main feature of these curves, as pointed out in the reference paper, is that the use of finite-wing coefficients is very influential at Mach numbers near unity but, as would be expected, becomes less so as the Mach number is increased. At $M = 10/9$, for the particular wing analyzed, the flutter speed obtained by using two-dimensional coefficients is about 62 percent of that obtained by using rectangular-wing section coefficients, whereas at $M = 10/6$ it is about 95 percent. For comparison at $M = 1.3$ the experimental value for model B-1 and the results of using two-dimensional coefficients with and without structural damping and total rectangular-wing coefficients in a representative-section analysis are included in figure 4. The values plotted in figure 4 at $M = 1.3$ are, of course, also given for model B-1 in figure 3(c). As may be noted in figure 4, the result obtained by using rectangular-wing section coefficients in a Rayleigh analysis is in excellent agreement with experiment.

CONCLUDING REMARKS

The results of applying four methods of flutter analysis to a series of twelve wings have been presented and discussed. The wings in question, which were fluttered previously at a Mach number of 1.3 in the Langley supersonic flutter apparatus, had aspect ratios ranging from 3.00 to 4.55 and various profile shapes, masses, and stiffness properties. The four methods of analysis, which are derivable from a general Rayleigh type of analysis, are: section coefficients for a pitching and translating wing in a Rayleigh type of analysis, two-dimensional coefficients in a Rayleigh type of analysis, total coefficients for a pitching and translating rectangular wing in a representative-section analysis, and two-dimensional coefficients in a representative-section analysis. Each of the four analyses involved two degrees of freedom, namely, first bending and first torsion of a cantilever wing. The section and total aerodynamic coefficients for rectangular wings that were used are those that were developed, for wing pitching and vertical translation, to the seventh power of the frequency in NACA TN 3076.

Previous analyses of the flutter of unswept wings of low aspect ratio in supersonic flow have customarily involved the use of aerodynamic coefficients for two-dimensional flow. The present paper shows that the use of aerodynamic coefficients for rigid-body motions of a wing, namely pitching and vertical translation, derived on the basis of three-dimensional flow leads, at least in the low supersonic speed range, to a significant improvement in the correlation of theory and experiment.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 13, 1954.

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TABLE I

PARAMETERS FOR WINGS TESTED FOR FLUTTER AT A MACH NUMBER OF 1.3

Model designation	A-1	B-1	B-2	B-3	B-4	B-5	C-1	C-2	D-1	E-1	F-1	G-1
Description of model section . . .	NACA 65-007	NACA 16-010	NACA 16-010	NACA 16-010	NACA 16-010	NACA 16-010	8-percent-thick, circular-arc	8-percent-thick, circular-arc	Modified circular-arc, 4.74-percent-thick	3-percent-thick, circular-arc	3-percent-thick, double-wedge	Modified circular-arc, 5-percent-thick
Aspect ratio, A . .	4.00	4.53	4.53	4.53	4.53	3.72	3.96	3.96	3.38	4.52	4.55	3.00
Elastic axis, x_0 . .	0.413	0.341	0.396	0.396	0.442	0.395	0.480	0.5155	0.570	0.387	0.452	0.475
x_α	0.156	0.350	0.300	0.328	0.250	0.350	0.100	0.0770	0.180	0.478	0.226	0.120
$1/\kappa$	64.9	93.5	108.1	113.1	113.3	130	67.1	74.1	53.5	267.3	150.8	51.7
r_α^2	0.28	0.39	0.38	0.40	0.37	0.37	0.250	0.255	0.275	0.310	0.29	0.27
ω_h/ω_α	0.48	0.585	0.645	0.653	0.57	0.64	0.432	0.435	0.614	0.308	0.215	0.606

TABLE II

COMPARISON OF CALCULATED AND EXPERIMENTAL FLUTTER PARAMETERS

(a) $V/b\omega$

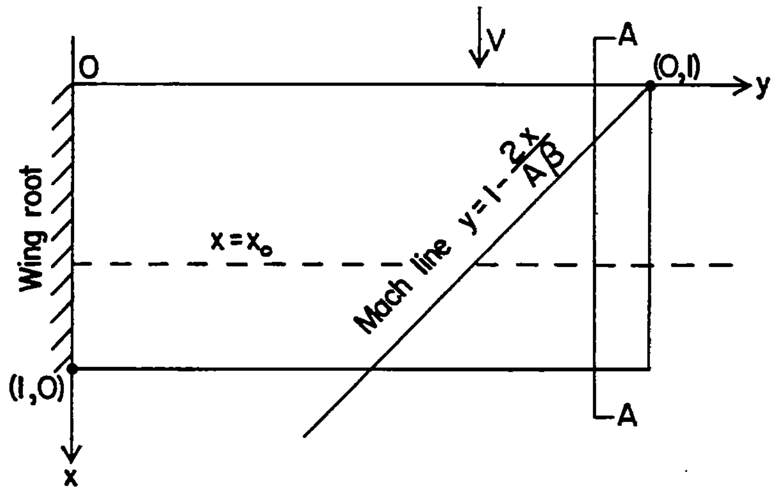
Model	Values of $V/b\omega$				
	Experiment	Rayleigh analysis		Representative-section analysis	
		Rectangular-wing section coefficients	Two-dimensional coefficients	Rectangular-wing total coefficients	Two-dimensional coefficients
A-1	10.15	8.96	4.63	7.09	4.81
B-1	9.98	9.10	6.38	7.89	6.98
B-2	10.31	8.45	7.02	7.39	7.17
B-3	10.20	8.87	7.52	7.78	7.49
B-4	10.40	9.10	7.46	8.10	7.45
B-5	10.03	9.60	7.97	8.23	7.85
C-1	9.715	8.78	5.45	7.16	5.45
C-2	9.92	8.90	5.90	7.29	5.93
D-1	9.04	7.10	4.33	5.97	4.50
E-1	19.13	18.27	13.48	16.05	14.32
F-1	19.61	14.45	8.12	11.65	8.55
G-1	7.71	6.93	4.65	5.19	4.64

(b) ω/ω_α

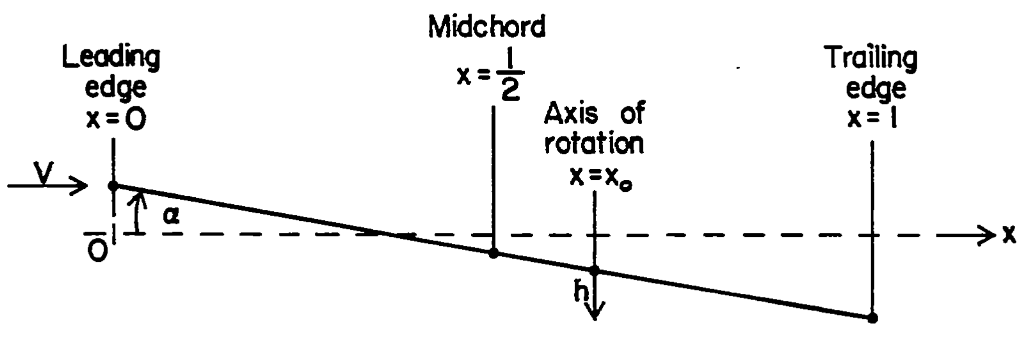
Model	Values of ω/ω_α				
	Experiment	Rayleigh analysis		Representative-section analysis	
		Rectangular-wing section coefficients	Two-dimensional coefficients	Rectangular-wing total coefficients	Two-dimensional coefficients
A-1	0.648	0.796	0.993	0.868	0.980
B-1	.822	.909	1.078	.905	1.012
B-2	.847	.823	.901	.811	.836
B-3	.870	.821	.862	.804	.825
B-4	.719	.776	.858	.787	.828
B-5	.840	.790	.841	.778	.785
C-1	.518	.695	.880	.764	.871
C-2	.51	.669	.828	.734	.821
D-1	.798	.823	1.031	.865	1.010
E-1	.868	.834	.953	.859	.905
F-1	.531	.718	.935	.781	.906
G-1	.718	.734	.837	.764	.822

(c) $V/b\omega_\alpha$

Model	Values of $V/b\omega_\alpha$				
	Experiment	Rayleigh analysis		Representative-section analysis	
		Rectangular-wing section coefficients	Two-dimensional coefficients	Rectangular-wing total coefficients	Two-dimensional coefficients
A-1	6.59	7.13	4.60	6.20	4.71
B-1	8.21	8.27	6.88	7.18	7.07
B-2	8.74	6.95	6.32	6.00	6.00
B-3	8.91	7.28	6.46	6.27	6.18
B-4	7.44	7.06	6.40	6.39	6.17
B-5	8.42	7.58	6.703	6.41	6.16
C-1	5.02	6.10	4.80	5.49	4.75
C-2	5.055	5.95	4.88	5.37	4.87
D-1	7.23	5.84	4.47	5.16	4.55
E-1	16.7	15.20	12.85	13.75	12.96
F-1	10.35	10.38	7.58	9.04	7.75
G-1	5.54	5.09	3.89	3.97	3.81



(a) Plan form (xy-plane).



(b) Section A-A.

Figure 1.- Illustration of coordinate system and two degrees of freedom α and h.

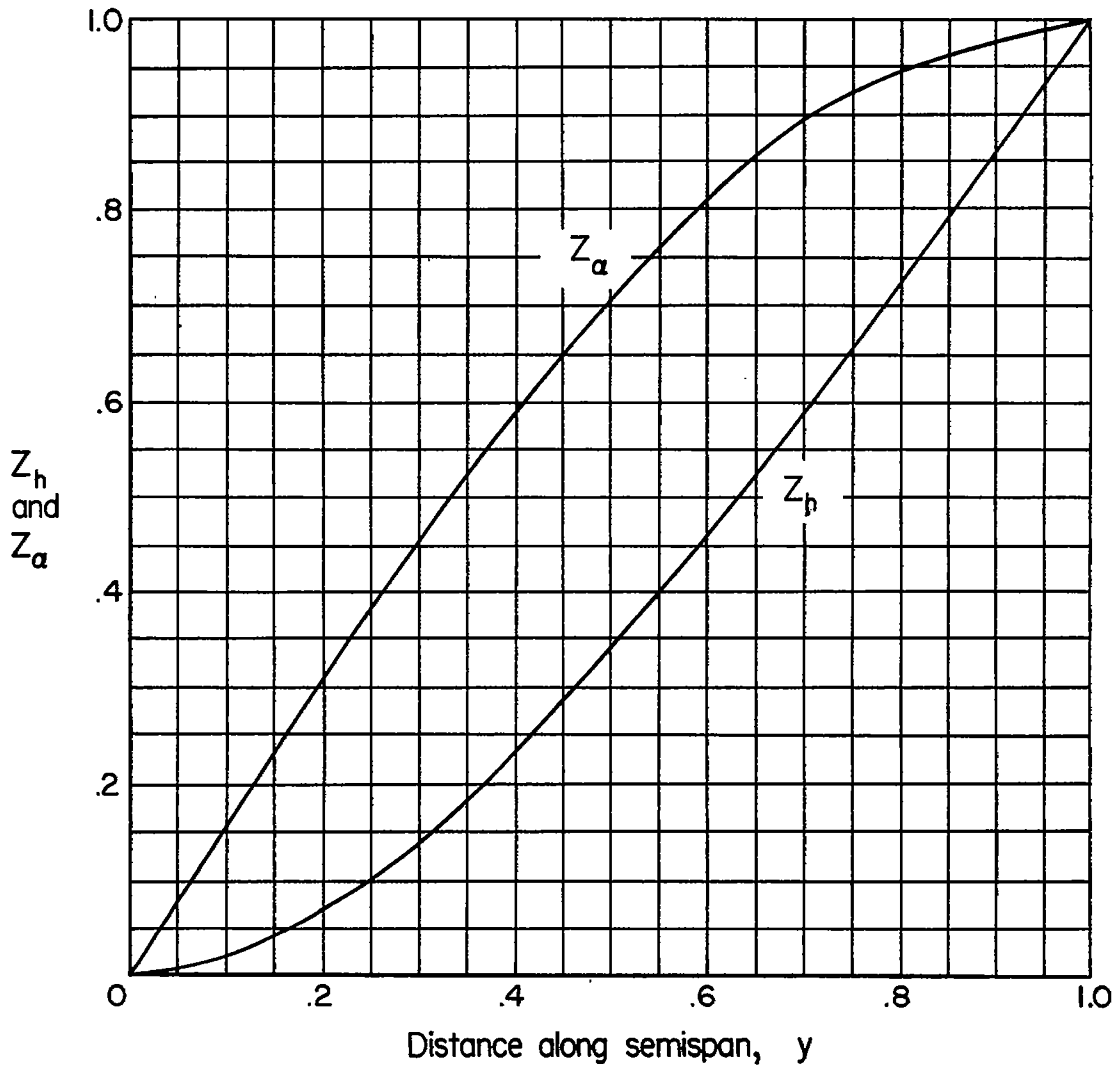
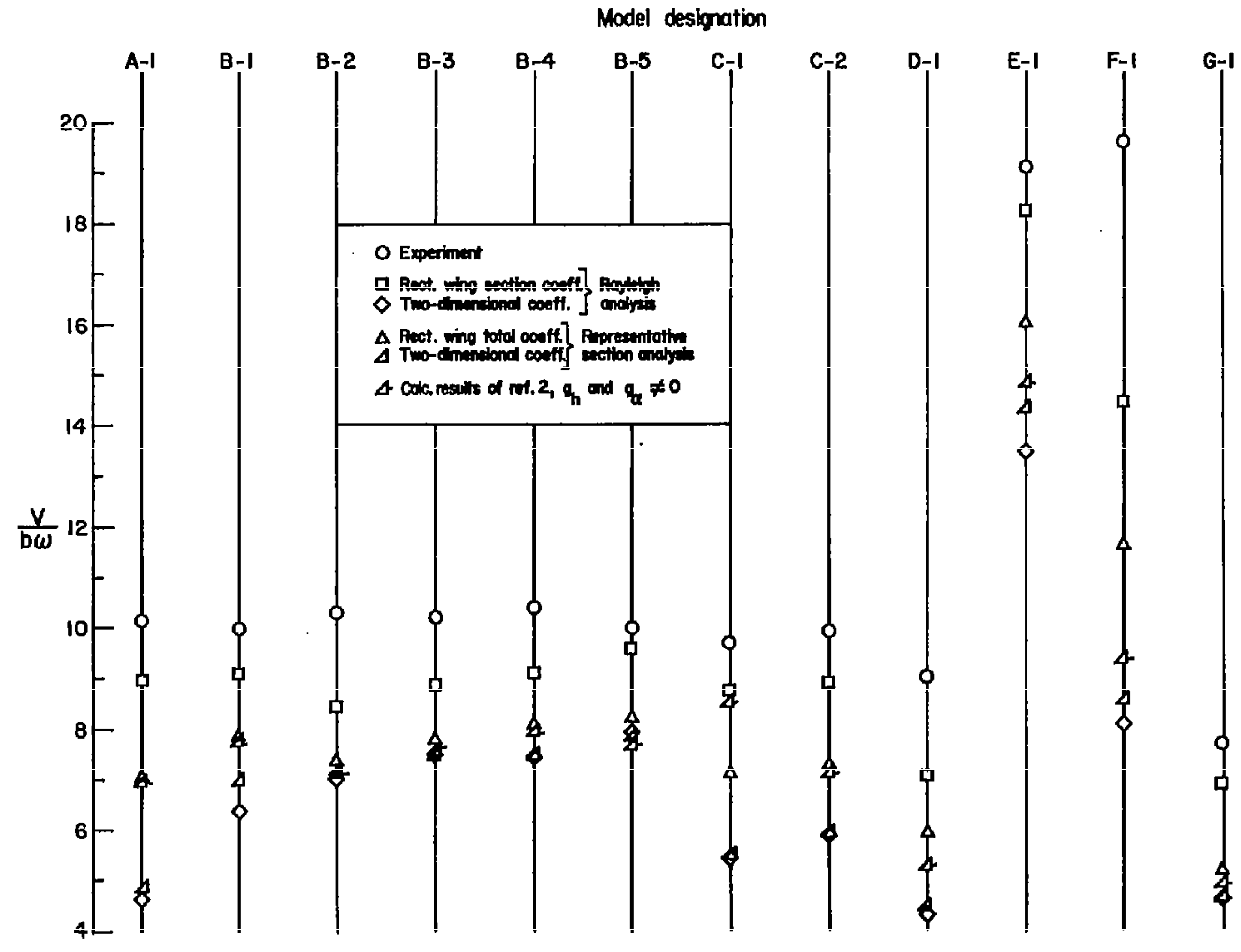
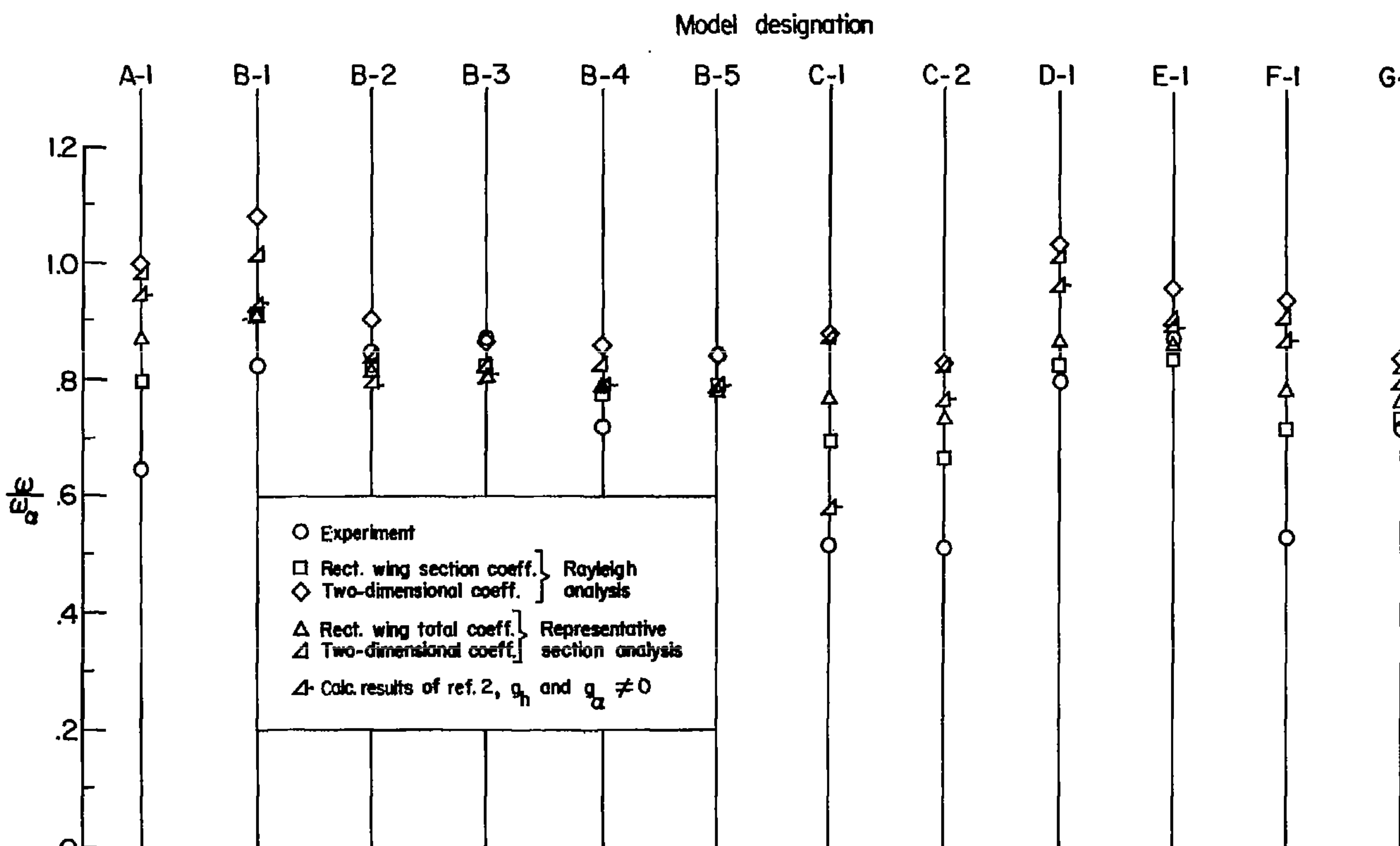


Figure 2.- Uncoupled first bending mode shape Z_h and first torsion mode shape Z_α for a uniform cantilever wing.



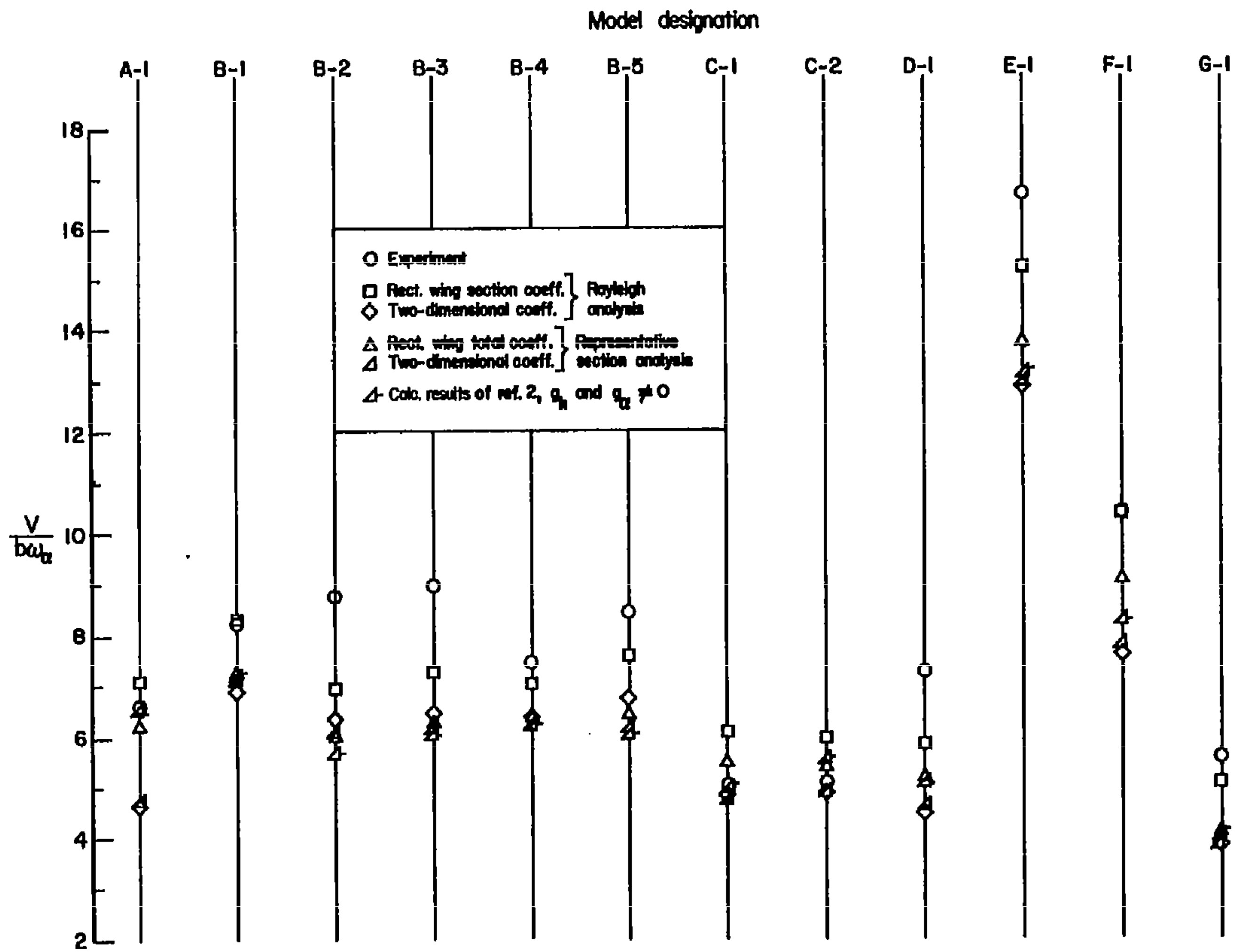
(a) Reduced flutter speed, $\frac{V}{b\omega}$.

Figure 3.- Comparison of calculated and experimental flutter characteristics for twelve wing models described in table I.



(b) Ratio of flutter frequency to torsional frequency, $\frac{\omega}{\omega_\alpha}$.

Figure 3.- Continued.



(c) Flutter speed coefficient, $\frac{V}{b\omega_\alpha}$.

Figure 3.- Concluded.

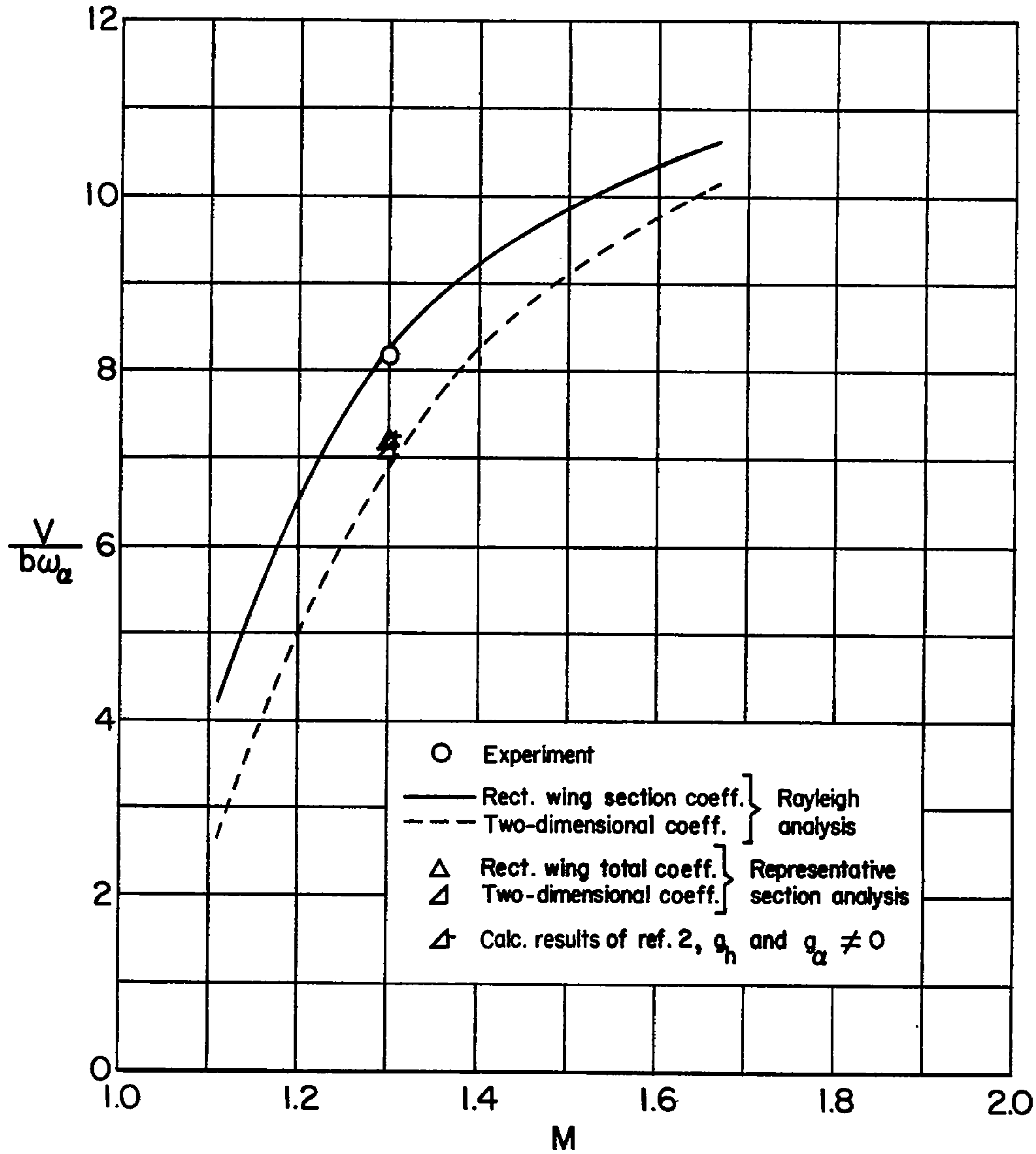


Figure 4.- Flutter-speed coefficients plotted against Mach number for model B-1 of table I. $\left(A = 4.53;\ \frac{1}{\kappa} = 95.3;\ x_o = 0.341;\ x_\alpha = 0.350;\ r_\alpha{}^2 = 0.39;\ \text{and}\ \frac{\omega_h}{\omega_\alpha} = 0.583.\right)$

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